UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

MATHEMATICS (SYLLABUS D) 4024/02

Paper 2

October/November 2004

2 hours 30 minutes

Additional Materials: Answer Booklet/Paper
Electronic calculator
Geometrical instruments
Graph paper (2 sheets)
Mathematical tables (optional)

READ THESE INSTRUCTIONS FIRST

Write your answers and working on the separate Answer Booklet/Paper provided.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Section A
Answer **all** questions.

Section B
Answer any **four** questions.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 100.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

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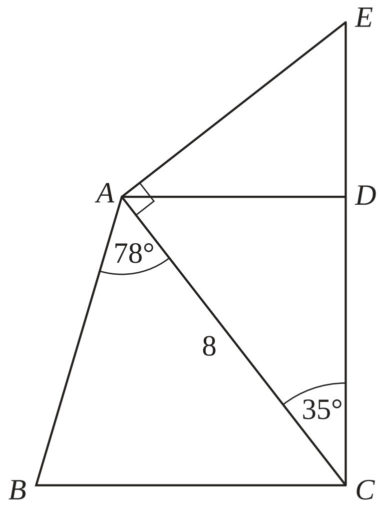
UNIVERSITY of CAMBRIDGE
International Examinations

Section A [52 marks]

Answer **all** the questions in this section.

1



The diagram represents some beams which support part of a roof.

AD and *BC* are horizontal and *CDE* is vertical.

$AC = 8$ metres, $B\hat{A}C = 78°$, $A\hat{C}D = 35°$ and $C\hat{A}E = 90°$.

Calculate the length of the beam

(a) *AD*, [2]

(b) *CE*, [2]

(c) *AB*. [3]

2 The points A, B and C are (9, 8), (12, 4) and (4, –2) respectively.

(a) Find

(i) the gradient of the line through A and B, [1]

(ii) the equation of the line through C which is parallel to AB. [2]

(b) Calculate the length of the line segment

(i) AB, [1]

(ii) BC. [1]

(c) Show that AB is perpendicular to BC. [1]

(d) Calculate the area of triangle ABC. [1]

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3 Mr Smith bought three companies, Alpha, Beta and Gamma, for a total of $80 000 000.

The amounts he paid for these companies were in the ratio 4 : 5 : 7.

(a) Calculate how much he paid for each company. [2]

(b) When he sold the companies, he made a profit of 12% on the $80 000 000 he paid for them.

Calculate the profit he made on the sale of the three companies. [1]

(c) When he sold the companies, he made a profit of 25% on Alpha and a loss of 10% on Beta.

Calculate

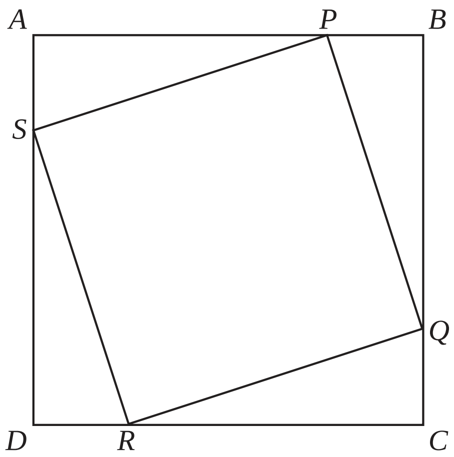
(i) the profit he made on Alpha, [1]

(ii) the percentage profit that he made on Gamma. [3]

(d) When the previous owner, Mr Jones, sold the companies to Mr Smith for $80 000 000, he made a profit of 60%.

Calculate the total amount Mr Jones had paid for the companies. [2]

4



In the diagram, $ABCD$ is a square.

Points P, Q, R and S lie on AB, BC, CD and DA so that $AP = BQ = CR = DS$.

(a) Giving all your reasons, show that

(i) $PB = QC$, [2]

(ii) triangle BPQ is congruent to triangle CQR, [3]

(iii) $P\hat{Q}R$ is a right angle. [2]

(b) Write down two reasons to show that $PQRS$ is a square. [2]

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5 **(a)** It is given that $S = \dfrac{n(a + l)}{2}$.

(i) Find the value of S when $n = 20$, $a = -5$ and $l = 17$. [1]

(ii) Express l in terms of S, n and a. [2]

(b) Solve the equations

(i) $5t^2 = 12t$, [2]

(ii) $\dfrac{y - 1}{8} = \dfrac{2}{y - 1}$. [3]

(c) Solve the equation $3x^2 + 9x + 5 = 0$, giving both answers correct to two decimal places. [4]

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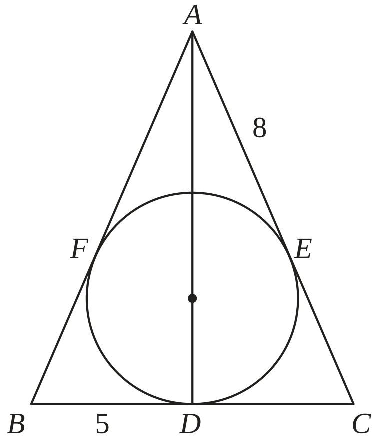
6 **(a)**



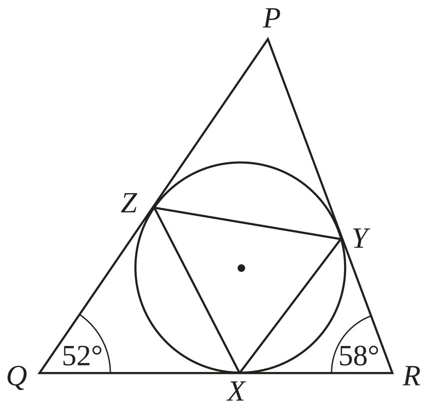
The diagram shows a circle which passes through *D*, *E* and *F*.
AFB, *BDC* and *CEA* are tangents to the circle.
D is the midpoint of *BC*.

Given that $BD = 5$ cm and $AE = 8$ cm, find

(i) *EC*, [1]

(ii) $C\hat{A}D$. [2]

(b)



The diagram shows a circle which passes through *X*, *Y* and *Z*.
PZQ, *QXR* and *RYP* are tangents to the circle.

Given that $P\hat{Q}R = 52°$ and $Q\hat{R}P = 58°$, calculate

(i) $Q\hat{P}R$, [1]

(ii) $Q\hat{Z}X$, [2]

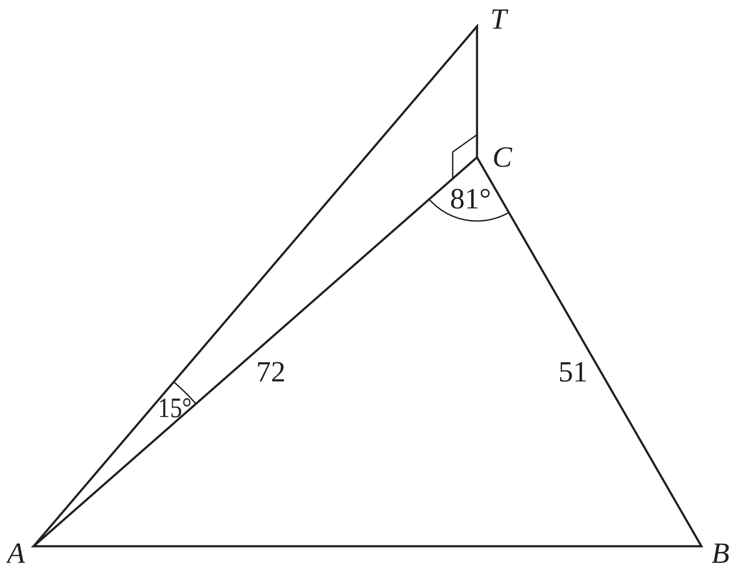
(iii) $Z\hat{X}Y$. [2]

Section B [48 marks]

Answer **four** questions in this section.

Each question in this section carries 12 marks.

7



Three paths, *AB*, *BC* and *CA*, run along the edges of a horizontal triangular field *ABC*.
BC = 51 m, *AC* = 72 m and angle *ACB* = 81°.

(a) Calculate the length of *AB*. [4]

(b) Calculate the area of the field *ABC*. [2]

(c) Calculate the shortest distance from *C* to *AB*. [2]

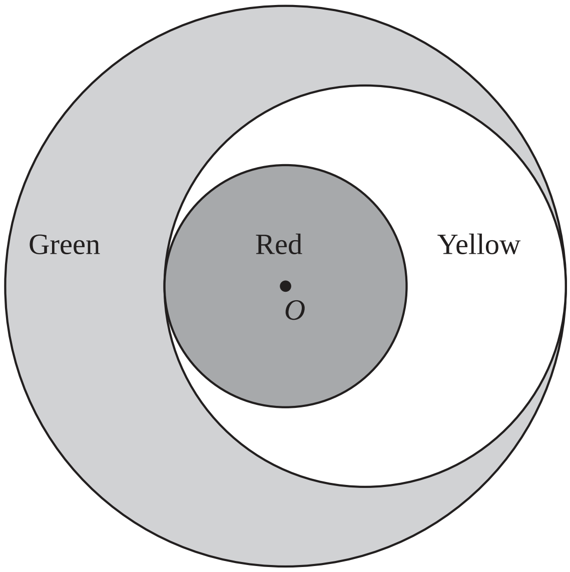
(d) A vertical tree, *CT*, has its base at *C*.
The angle of elevation of the top of the tree from *A* is 15°.

Calculate the height of the tree. [2]

(e) John measured the largest angle of elevation of the top of the tree as seen from the path *AB*.

Calculate this angle of elevation. [2]

8



The diagram shows the design of a company symbol.

It consists of three circles.
The smallest circle has centre O and radius $2x$ centimetres.
The largest circle has centre O and radius $2y$ centimetres.
The third circle touches both the other two circles as shown.
The three regions formed are coloured red, yellow and green as shown.

(a) Explain fully why the radius of the third circle is $(x + y)$ centimetres. [2]

(b) Write down, in terms of π, x and y, expressions for the area of the region that is coloured

(i) yellow, [1]

(ii) green. [1]

(c) The area of the green region is twice the area of the yellow region.
Use this information to write down an equation involving x and y, and show that it simplifies to

$$y^2 - 6xy + 5x^2 = 0.$$ [3]

(d) **(i)** Factorise $y^2 - 6xy + 5x^2$. [1]

(ii) Solve the equation $y^2 - 6xy + 5x^2 = 0$, expressing y in terms of x. [2]

(e) Calculate the fraction of the design that is coloured yellow. [2]

[Turn over

9 Three integers, a, b and c, are such that $a < b < c$.
The three integers are said to form a Pythagorean Triple if $c^2 = a^2 + b^2$ or $c^2 - b^2 = a^2$.

For example
3, 4, 5 form a Pythagorean Triple because $5^2 - 4^2 = (5 - 4)(5 + 4) = 1 \times 9 = 9 = 3^2$
and 5, 12, 13 form a Pythagorean Triple because $13^2 - 12^2 = (13 - 12)(13 + 12) = 1 \times 25 = 25 = 5^2$.

(a) In the same way, show that 7, 24 and 25 form a Pythagorean Triple. [1]

(b) Form a Pythagorean Triple

(i) in which the last two integers are 40 and 41, [1]

(ii) in which the first integer is 11. [1]

(c) **(i)** Simplify $(n + 1)^2 - n^2$. [1]

(ii) Hence form a Pythagorean Triple in which the first integer is 101. [2]

(d) It is also possible to form Pythagorean Triples in which the last two integers differ by 2.

For example
8, 15, 17 form a Pythagorean Triple because $17^2 - 15^2 = (17 - 15)(17 + 15) = 2 \times 32 = 64 = 8^2$.

(i) Copy and complete the following statements:

… , 35, 37 form a Pythagorean Triple because $37^2 - 35^2 = (\quad)(\quad) = \ldots\ldots \times \ldots\ldots = \ldots\ldots = \ldots\ldots$. [1]

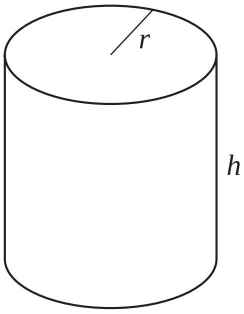
16, … , … form a Pythagorean Triple because $\ldots\ldots - \ldots\ldots = (\quad)(\quad) = 2 \times \ldots\ldots = \ldots\ldots = 16^2$. [2]

(ii) Simplify $(4n^2 + 1)^2 - (4n^2 - 1)^2$ and hence express it as a perfect square. [2]

(iii) Form a Pythagorean Triple in which the first integer is 400 and the other two integers differ by 2. [1]

10 Answer the whole of this question on a sheet of graph paper.

A solid cylinder of radius r centimetres and height h centimetres has a volume of $100\pi\ \text{cm}^3$.



(a) **(i)** Show that $h = \dfrac{100}{r^2}$. [1]

(ii) The cylinder has a total surface area of πy square centimetres.

Show that $y = 2r^2 + \dfrac{200}{r}$. [1]

(b) The table below shows some values of r and the corresponding values of y, correct to the nearest whole number.

r	1	1.5	2	3	4	5	6
y	202	138	108	85	82	90	p

(i) Find the value of p. [1]

(ii) Using a scale of 2 cm to represent 1 cm, draw a horizontal r-axis for $1 \leqslant r \leqslant 6$.
Using a scale of 2 cm to represent 20 cm^2, draw a vertical y-axis for $70 \leqslant y \leqslant 220$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the values of r for which $y = 100$. [2]

(d) By drawing a tangent, find the gradient of the graph at the point where $r = 2$. [2]

(e) Use your graph to find

(i) the value of r for which y is least, [1]

(ii) the smallest possible value of the total surface area of the cylinder. [1]

11 Answer the whole of this question on a sheet of graph paper.

The table below shows the marks obtained in tests of English and Mathematics by 140 students.

Mark (x)	Number of candidates	
	English	Mathematics
$0 < x \leqslant 20$	4	10
$20 < x \leqslant 40$	26	20
$40 < x \leqslant 60$	50	30
$60 < x \leqslant 80$	56	55
$80 < x \leqslant 100$	4	25

(a) Copy and complete the cumulative frequency table below.

Mark (x)	Number of candidates	
	English	Mathematics
$x = 0$	0	0
$x \leqslant 20$	4	
$x \leqslant 40$		
$x \leqslant 60$		
$x \leqslant 80$		
$x \leqslant 100$	140	

[2]

(b) Using a scale of 2 cm to represent 20 marks, draw a horizontal x-axis for $0 \leqslant x \leqslant 100$.
Using a scale of 2 cm to represent 20 pupils, draw a vertical axis for values from 0 to 140.
On your axes, draw and label both smooth cumulative frequency curves to illustrate this information. [3]

(c) Use your curves to find

(i) the upper quartile mark for English, [1]

(ii) the interquartile range for English, [1]

(iii) the median mark for English and the median mark for Mathematics. [1]

(d) State, with a reason, which you think is the easier test. [1]

(e) One student is chosen at random.
It may be assumed that the marks gained in the two subjects are independent.

Expressing each answer as a fraction in its lowest terms, calculate the probability that the student gains

(i) more than 60 marks on both papers, [1]

(ii) more than 80 marks on one paper, but not on the other. [2]

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